## CONTENTS

PREFACE ..... v
Chapter 1 Linear Functions ..... 1
1.1 Simultaneous equations ..... 2
1.2 Numbers, notation, and geometry ..... 13
1.3 Linear transformations ..... 28
1.4 Linear subspaces of $R^{n}$ ..... 40
1.5 Rank + nullity $=$ dimension ..... 53
1.6 Invertible matrices ..... 59
1.7 Eigenvectors and change of basis ..... 76
1.8 Complex numbers ..... 85
1.9 Space geometry ..... 93
1.10 Abstract notions of linearity ..... 105
1.11 Inner products ..... 110
Miscellaneous problems ..... 121
Chapter 2 Notions of Calculus ..... 126
2.1 Convergence of sequences ..... 129
2.2 Series ..... 137
2.3 Tests for convergence ..... 145
2.4 Convergence in $R^{n}$ ..... 153
2.5 Continuity ..... 159
2.6 Calculus of one variable ..... 165
2.7 Multiple integration ..... 173
2.8 Partial differentiation ..... 185
2.9 Improper integrals ..... 195
2.10 The space of continuous functions ..... 201
2.11 The fixed point theorem ..... 211
2.12 Summary ..... 219
Miscellaneous problems ..... 222
Chapter 3 Ordinary Differential Equations ..... 227
3.1 Differentiation ..... 228
3.2 Taylor's formula ..... 240
3.3 Differential equations ..... 250
3.4 Some techniques for solving equations ..... 259
3.5 Existence theorems ..... 266
3.6 Linear differential equations ..... 275
3.7 Second-order linear equations ..... 289
3.8 Summary ..... 298
Miscellaneous problems ..... 302
Chapter 4 Curves ..... 307
4.1 Parametrization of curves ..... 313
4.2 Arc length ..... 331
4.3 Local geometry of curves ..... 349
4.4 Curves in space ..... 359
4.5 Varying a curve in the plane ..... 365
4.6 Vector fields and fluid flows ..... 380
4.7 Summary ..... 393
Miscellaneous problems ..... 397
Chapter 5 Series of Functions ..... 400
5.1 Convergence ..... 401
5.2 The fundamental theorem of algebra ..... 406
5.3 Constant coefficient linear differential equations ..... 410
5.4 Solutions in series ..... 414
5.5 Power series ..... 421
5.6 Complex differentiation ..... 428
5.7 Differential equations with analytic coefficients ..... 434
5.8 Infinitely flat functions ..... 441
5.9 Summary ..... 445
Miscellaneous problems ..... 448
Chapter 6 Functions on the Circle (Fourier Analysis) ..... 452
6.1 Approximation by trigonometric polynomials ..... 453
6.2 Laplace's equation ..... 467
6.3 Fourier sine and cosine series ..... 476
6.4 The one-dimensional wave and heat equations ..... 482
6.5 The geometry of Fourier expansions ..... 495
6.6 Differential equations on the circle ..... 503
6.7 Taylor series and Fourier series ..... 509
6.8 Summary ..... 512
Miscellaneous problems ..... 517
Chapter 7 Line Integrals and Green's Theorem ..... 525
7.1 The differential ..... 527
7.2 Coordinate changes ..... 534
7.3 Differential forms ..... 547
7.4 Work and conservative fields ..... 552
7.5 Integration of differential forms ..... 560
7.6 Applications of Green's theorem ..... 574
7.7 The Cauchy integral formula ..... 584
7.8 Summary ..... 602
Miscellaneous problems ..... 607
Chapter 8 Potential Theory in Three Dimensions ..... 611
8.1 Divergence and the equation of continuity ..... 613
8.2 Curl and rotation ..... 624
8.3 Surfaces ..... 635
8.4 Surface integrals and Stokes' theorem ..... 657
8.5 The divergence theorem ..... 666
8.6 Dirichlet's principle ..... 674
8.7 Summary ..... 686
Miscellaneous problems ..... 690
ANSWERS TO SELECTED EXERCISES ..... 694
INDEX ..... 723

